Slightly Lengthy Problems

1) Show that pressure $P = \frac{1}{3}\int vpf(p)4\pi p^2 dp$, assuming particles striking the walls of a container to generate pressure are classical. Use Maxwell's distribution to show that pressure $P \propto k_B T$

2) The pressure due to degenerate electrons is given by $P = \frac{8\pi}{3h^3}\int_0^{p_F}\frac{p^4C^2}{\sqrt{p^2C^2+m_e^2C^4}}dp$. Solve this equation assuming $p = m_e C\, sin(hx)$

3) Lane-Emden equation is $\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n$; where n=polytropic index. This equation was obtained under the following change of variables.

    1) The pressure $P = K\rho^{(1+\frac{1}{n})}$
    2) $\rho = \rho_c\theta^n$ and
    3) $r = a\xi$.

    Here ρ, ρ_c are the density and central density respectively and r=radius, $a = \sqrt{[\frac{(n+1)K\rho_c^{\frac{1-n}{n}}}{4\pi G}]}$.

    Solve Lane-Emden equation for n=0 and interpret the solution.

Very Easy Problems

4) The sun has a rotation period of 27 days. If sun became a 1) white dwarf star 2) Neutron star, what will be their rotation period? Assume typical values.
5) Estimate the maximum possible radius of pulsar CP 1919, if the mean pulse width of the pulses is 0.04 seconds. Assume *causality principle*.
6) Crab pulsar has a pulse period of 33.5 milli seconds. This pulse period is increasing every day by about 38 nano seconds. It is generally assumed that the rotational K.E of crab pulsar (= $\frac{1}{2}I\omega^2$) is the energy source that drives the radiation. If the pulsar period increases due to this loss of energy, how long can the crab pulsar continue to be a pulsar?
7) A pulsar's magnetic field at the poles is given by the equation $B_{pole} = \sqrt{\frac{3\mu_0 C^3 I}{8\pi^2 R^6 sin^2\alpha}P\dot{P}}$ if the pulsar is assumed to be a rotating magnetic dipole. Here I= moment of Inertial, R=radius of the pulsar and α = angle between the rotation angle and magnetic dipole axis. Assuming α to be 90°, estimate the pole strength of crab pulsar. As before, P=33.5 milli seconds, $\dot{P}$ = 38 nano seconds/day. Take typical values for the mass and radius of a pulsar.